Statistics

Lecture 7

Feb 19-8:47 AM

SG 12

Odds vs Probability

odds in favor of event E are

$a : b$

\# of times E happens

\# of times $\bar{E}$ happens

Always simplify

If we flip a coin 20 times, and 6 times it lands tails.

$6 : 14 \Rightarrow \boxed{3:7}$

tails $\overline{\text{tails}}$

odds in favor of landing tails

Odds against landing tails $\Rightarrow 7:3$

Oct 11-8:06 AM

A box has 8 Red, 12 White, and 20 Blue balls.

1) P(select Red) $= \frac{8}{40} = \frac{1}{5}$

2) odds in favor of selecting a red ball

\# Red : \# $\overline{\text{Red}}$

8 : 32 → $\boxed{1:4}$

3) odds against selecting a red ball

$\boxed{4:1}$

4) odds in favor of selecting a Blue ball.

\# Blue : \# $\overline{\text{Blue}}$

20 : 20 → 1:1

Oct 11-8:11 AM

If odds in favor of event E are $a:b$,

then $P(E) = \frac{a}{a+b}$, $P(\overline{E}) = \frac{b}{a+b}$

Suppose odds in favor of event E are 4:21

1) odds against event E. $\boxed{21:4}$

2) $P(E) = \frac{4}{4+21} = \boxed{\frac{4}{25}}$

$= \boxed{.16}$

3) $P(\overline{E}) = \frac{21}{4+21} = \boxed{\frac{21}{25}}$

$= \boxed{.84}$

As you can see

$P(E) + P(\overline{E}) = 1$ ✓

Oct 11-8:16 AM

If $P(E)$ is given, then odds in favor of event E are $P(E) : P(\overline{E})$

Always Simplify

ex: Suppose $P(E) = .04$

1) $P(\overline{E}) = 1 - P(E) = 1 - .04 = \boxed{.96}$

2) odds in favor of event E are

$P(E) : P(\overline{E})$

$.04 : .96 \rightarrow \boxed{1 : 24}$

.04 $\boxed{\div}$ .96 $\boxed{\text{Math}}$ $\boxed{\text{1: ▸Frac}}$ $\boxed{\text{Enter}}$ $\frac{1}{24}$

3) odds against event E are $\boxed{24 : 1}$

Oct 11-8:21 AM

Given $P(E) = .85$

1) $P(\overline{E}) = 1 - P(E) = 1 - .85 = \boxed{.15}$

2) odds in favor of event E.

$P(E) : P(\overline{E})$

$.85 : .15 \rightarrow \boxed{17 : 3}$

.85 $\boxed{\div}$ .15 $\boxed{\text{Math}}$ $\boxed{\text{1: ▸Frac}}$ $\boxed{\text{Enter}}$

$\frac{17}{3}$

3) odds against event E.

$\boxed{3 : 17}$

Oct 11-8:27 AM

Odds & Gambling

True odds

a : b

$ bet — $ Net profit

Suppose odds in favor of Los Angles Rams win the Super bowl this year are

1 : 499

Place $1 on Rams to win the Super bowl

and if they win it, You get $500 in return. Net profit $500 − $1 = $499

Not True odds

Vegas does it with different notation

+150 → $100 bet, Net profit $150

−225 → $225 bet, Net Profit $100

Oct 11-8:32 AM

Multiplication Rule

Key word AND

Multiple Action Event

P(A and B)

A happens then B happens

Independent Events

Outcome of one event does not change the prob. of next event.

P(B) = .5 P(G) = .5

Rolling a fair die

$P(\text{Get } 6) = \frac{1}{6}$ on each roll.

$P(\text{Ace}) = \frac{1}{13}$ if You draw a card again with replacement

$P(\text{Ace}) = \frac{1}{13}$ on each draw

Multiple-choice exam

Each question has 4 choices.

Only one is a correct choice.

Making Random guesses.

$P(\text{Guess Correctly}) = \frac{1}{4}$ on each question

Oct 11-8:40 AM

If A & B are independent events, then
$P(A \text{ and } B) = P(A) \cdot P(B)$

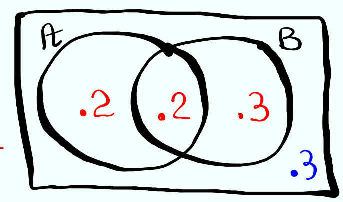
ex: $P(A) = .4$, $P(B) = .5$, A & B are independent events

1) $P(A \text{ and } B) = P(A) \cdot P(B) = (.4)(.5) = \boxed{.2}$

2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= .4 + .5 - .2 = \boxed{.7}$
Addition Rule

3) Draw Venn Diagram

Total = 1



Oct 11-8:47 AM

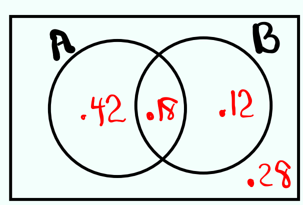
Suppose $P(A) = .6$, $P(B) = .3$ A & B are Independent Events

1) $P(\bar{A}) = 1 - P(A) = 1 - .6 = \boxed{.4}$

2) $P(A \text{ and } B) = P(A) \cdot P(B) = (.6)(.3) = \boxed{.18}$

3) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= .6 + .3 - .18 = \boxed{.72}$
Addition Rule

4) Draw Venn Diagram



Total = 1

Oct 11-8:54 AM

A loaded Coin is tossed twice.

$P(\text{Tails}) = .7$

$P(\text{Heads}) = .3$

TT TH HT HH

$\underbrace{\text{TT TH HT HH}}$

Sample Space

A complete list of all possible outcomes

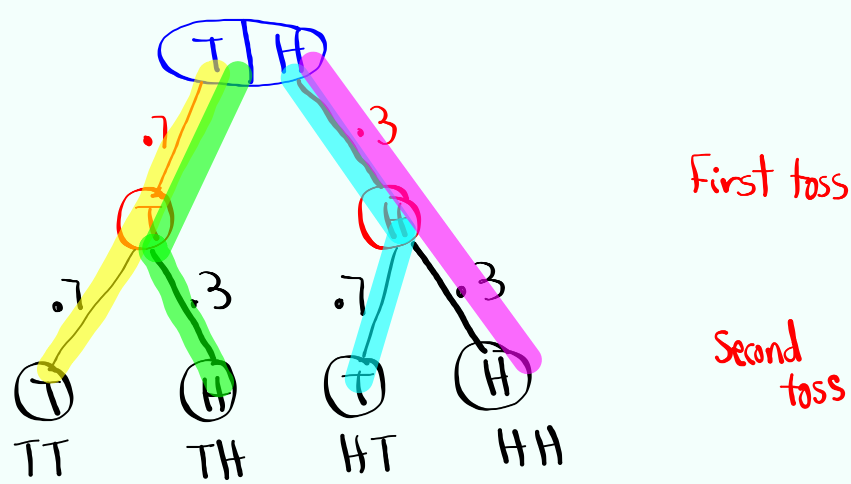
$P(TT) = (.7)(.7) = .49$

$P(TH \,\&\, HT) = 2(.3)(.7) = .42$

$P(HH) = (.3)(.3) = .09$

Oct 11-8:59 AM

Multiplication with Tree Diagram



$P(TT) = (.7)(.7) = .49$

$P(HH) = (.3)(.3) = .09$

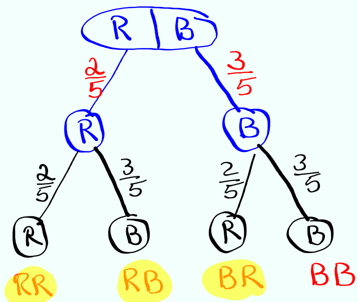
$P(TH) = (.7)(.3) = .21$

$P(HT) = (.3)(.7) = .21$

Oct 11-9:04 AM

A box has 2 Red & 3 Blue balls.

Select 2 balls with replacement



$P(RR) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = .16$

$P(\text{at least } 1 \text{ Red}) = 1 - P(\text{No Red})$

$= 1 - P(BB) = 1 - \frac{3}{5} \cdot \frac{3}{5}$

$= 1 - \frac{9}{25}$

$= \boxed{\frac{16}{25}} = \boxed{.64}$

Oct 11-9:09 AM

Dependent Events (SG 13)

outcome of one event changes the Prob. of next event.

when prob. changes → we have dependent events

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

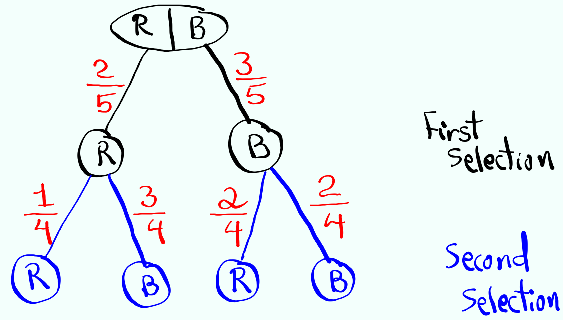
A happens, then B happens

Given

Oct 11-9:27 AM

A box has 2 Red & 3 Blue balls.

Select 2 balls, $\boxed{\text{No replacement}}$



$P(RR) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$

$P(BR) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$

$P(RB) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20}$

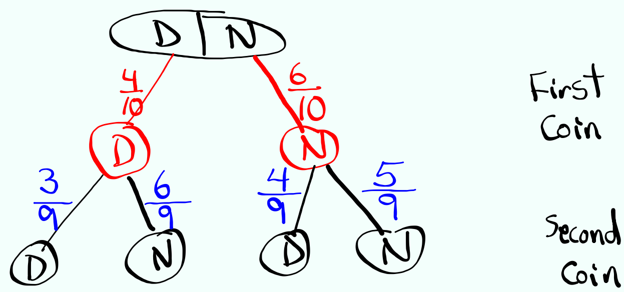
$P(BB) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$

$P(\text{at least 1 blue ball}) = 1 - P(\text{No blue ball})$
$= 1 - P(RR) = 1 - \frac{2}{20} = \frac{18}{20} = \frac{9}{10}$

Oct 11-9:31 AM

A piggy bank has 4 dimes and 6 nickels.

Take 2 Coins, No replacement.



$P(20¢) = P(DD) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90}$

$P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{6}{10} \cdot \frac{4}{9} = \frac{48}{90}$

$P(10¢) = P(NN) = \frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90}$

Oct 11-9:38 AM

A deck of playing cards has 40 cards, 18 Red, 10 face, and 3 aces.

Take 3 cards, $\boxed{\text{No replacement}}$

$P(\text{All Red}) = \frac{18}{40} \cdot \frac{17}{39} \cdot \frac{16}{38} = \boxed{\frac{102}{1235}}$

$P(\text{All Black}) = \frac{22}{40} \cdot \frac{21}{39} \cdot \frac{20}{38} = \boxed{\frac{77}{494}}$

$P(\text{all selections are same color}) = \frac{102}{1235} + \frac{77}{494}$

RRR OR BBB

$= \boxed{\frac{31}{130}}$

$P(\text{they are not all same color}) = 1 - P(\text{Same})$

$= 1 - \frac{31}{130} = \frac{99}{130}$

P(at least 1 red color card)

RRR

Some R & Some B

BBB

$= 1 - P(\text{No red})$

$\uparrow$ Total Prob.

$= 1 - P(BBB) = 1 - \frac{77}{494} = \boxed{\frac{417}{494}}$

Oct 11-9:46 AM

4 Females 6 Males

Select 4 people

$P(\text{All Females}) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} \cdot \frac{1}{7} = \boxed{\frac{1}{210}}$

$P(\text{All Males}) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} \cdot \frac{3}{7} = \boxed{\frac{1}{14}}$

$P(\text{at least 1 Female}) = 1 - P(\text{No female}) = 1 - \frac{1}{14} = \frac{13}{14}$

$P(\text{at least 1 Male}) = 1 - P(\text{No Male}) = 1 - \frac{1}{210} = \frac{209}{210}$

FFFF

Some F & Some M

MMMM

Oct 11-9:58 AM

Dependent Events

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Given

If we divide by $P(A)$,

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Conditional Prob.

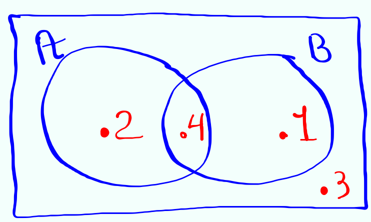
Oct 11-10:17 AM

$P(A) = .6$

$P(B) = .5$

$P(A \text{ and } B) = .4$

1) Venn Diagram



Total = 1

2) $P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B) = .6 + .5 - .4 = \boxed{.7}$

3) $P(\overbrace{B|A}^{\text{AND}}) = \dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{.4}{.6} = \boxed{\dfrac{2}{3}} = \boxed{.667}$

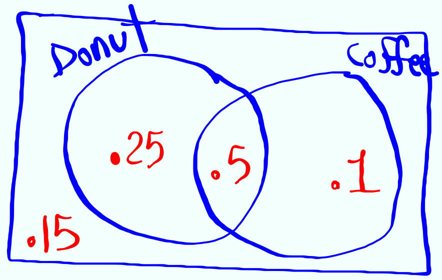
4) $P(\overbrace{A|B}^{\text{And}}) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{.4}{.5} = \boxed{.8}$

Oct 11-10:19 AM

$P(\text{Donut}) = .75$

$P(\text{Coffee}) = .6$

$P(\text{Donut and Coffee}) = .5$



Total = 1

$$P(\text{Coffee} \mid \underbrace{\text{Donut}}_{}) \overset{\text{And}}{} = \frac{P(\text{Coffee and Donut})}{P(\text{Donut})} = \frac{.5}{.75} \approx \boxed{.667}$$

$$P(\text{Donut} \mid \text{Coffee}) \overset{\text{AND}}{} = \frac{P(\text{Coffee and Donut})}{P(\text{Coffee})} = \frac{.5}{.6} \approx \boxed{.833}$$

Oct 11-10:25 AM

$P(A) = .5$

$P(B) = .6$

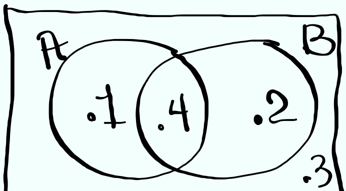
$P(B \mid A) = .8$

1) $P(A \text{ and } B)$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$.8 = \frac{P(A \text{ and } B)}{.5}$$

$$P(A \text{ and } B) = (.8)(.5) = \boxed{.4}$$



AND

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.4}{.6} = \frac{2}{3} \approx .667$$

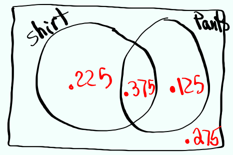
Oct 11-10:33 AM

P(Shirt) = .6 P(Shirt and Pants)

P(Pants) = .5 → .6 − .375 → .5 − .375

P(shirt | Pants) = .75

$$P(\overbrace{\text{shirt} \mid \text{Pants}}^{\text{AND}}) = \frac{P(\text{Shirt and Pants})}{P(\text{Pants})}$$



Total = 1

$$.75 = \frac{P(\text{shirt and Pants})}{.5}$$

Cross-Multiply

P(Shirt and Pants) = .75(.5)

$= \boxed{.375}$

$$P(\overbrace{\text{Pants} \mid \text{shirt}}^{\text{And}}) = \frac{P(S \,\&\, P)}{P(\text{Shirt})} = \frac{.375}{.6} = \boxed{.625}$$

SG 12 & SG 13

Oct 11-10:40 AM